An Efficient Approach for Mining of Outliers for Imperfectly Labeled Data

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Abstract —Detection of outliers is a significant problem in the context of data analysis. Many of these existing techniques assume that the instance can be classified as either normal or abnormal class. But, many real-life applications contain data which are uncertain in nature because of many errors or partial completeness. This uncertainty in data gives rise to imperfect labelling of the data. Thus, important challenge lies in handling the imperfectly labelled data to nullify the effect of uncertain data on the classifier. This paper proposes a modified Support Vector Data Description based approach to detect outliers for uncertain data. The proposed system executes in two steps. The first step deals with generating the pseudo-training dataset by calculating the likelihood values. The second step deals with incorporating the generated confidence score into the support vector data description training phase to generate a global classifier. Here, the effect of the instances with the least confidence score on the construction of the decision boundary is reduced.

Key Words — Anomaly Detection; Likelihood Values; Support vector data description

I. INTRODUCTION

Outlier detection refers to the problem of detecting and analyzing patterns in data that does not map to expected normal behavior. These patterns are often referred to as outliers, anomalies, surprise, exceptions, noise, defects, errors, damage, faults, aberrations, discordant observations, novelty, contaminants or peculiarities in different application domain [2]. Outlier detection has always been a widely researched problem. It finds immense use in a wide variety of application domains such as insurance, tax, military surveillance for enemy activities, credit card, fraud detection, fault detection in safety critical systems, intrusion detection for cyber security and many other areas [2]. Traditional outlier detection techniques typically assume that outliers are difficult or costly to obtain because of their rare occurrence. Hence, most of the previous techniques focus on modeling a representation of the normal data in order to identify outliers that do not fit the model. These outlier detection algorithms are mainly classified into four basic categories. Firstly, in Statistics-based algorithms [3], statistical techniques fit a statistical model to the given data and then apply a statistical inference test in order to determine if an incoming instance fits the model or not. Secondly, in Density based method [4] local outliers are identified by analyzing the distances to their nearest neighbors. Thirdly, Clustering based approaches [5] groups identical data instances into same clusters and considers clusters of smaller size as outliers. Fourthly, Model-based method [6] generates distinctive model from a set of training data instances to detect outliers which deviates from the model. In this category, Support Vector Data Descriptor (SVDD) [1, 7], determine a sphere around normal data with minimum volume. Another vital observation is that, data are uncertain in nature for many real-life applications. For example, the data points may correspond to objects, which are only vaguely specified due to data incompleteness, and are therefore considered uncertain in their representation; moreover, some new hardware technologies such as sensors usually collect large amounts of uncertain data due to sampling errors or instrument imperfections. Consequently, a labeled normal example corrupted by various errors or limitations of the underlying equipment always behaves like an outlier, even though the example itself may not be an outlier. This always makes the problem of outlier detection far more difficult from the perspective of data uncertainty. Therefore, it is worthwhile to develop techniques to refine the decision boundary of the distinctive classifier so as to improve the performance of outlier detection. The key challenge of handling of data with uncertainty in outlier detection is how to minimize the impact of this uncertain data on the learned distinctive classifier.

The remainder of paper is organized as follows: Section 2 discuss the related work done and its shortcoming. An overview of the proposed scheme is given in section 3. Section 4 discusses the expected results of the system. Finally, we conclude the paper.

II. LITERATURE SURVEY

A. Outlier Detection

Outlier detection deals with the problem of finding patterns in large data that do not conform to expected pattern or behavior. Lot of work has been done in this area which can be classified into the following four categories:

1) Statistics-based approaches: This tries to fit a statistical model to the given data and then it apply a statistical inference test in order to determine whether an unseen instance or record satisfies this model or not. The instances which have less probability of being generated from the learned model are declared as outliers. For example, we can assume the normal instance follow a certain data distribution like Gaussian distribution [8][9]. For this category, the main disadvantage lies in the assumption that the data is generated from a particular distribution.

2) Density-based approaches: Density-based approaches assume that normal data instances occur in dense neighborhoods, whereas outliers occur far from their closest neighbors [11]. One of the important methods is called LOF (local outlier factor).
LOF method assigns an outlier score or local outlier factor to any given data point, depending on its distances in the local neighborhood. But, in the case where data has normal instances with no enough close neighbors or if the data has outliers with enough close neighbors then the technique fails to label them correctly which further results in incorrect outliers.

3) Clustering-based approaches: Clustering-based methods [14, 15] is one of the highly researched method stream. After clustering, small clusters containing significantly fewer data points than other clusters are deemed as outliers. For clustering based techniques, the performance is highly dependent on the effectiveness of the clustering algorithm in the way of capturing the cluster structure of normal instances.

4) Model-based approaches: Model-based techniques are used to learn a model i.e. a classifier from a set of labeled data instances i.e. training and then to classify a test instance into one of the classes using the learnt model i.e. testing [16]. Model-based outlier detection approach operates in two-phase fashion. The training phase learns a classifier by using the available labeled training data. The testing phase classifies a test instance as normal or outlier using the classifier. In this approach, SVDD proposed by [17-21], has demonstrated the cluster structure of normal instances. Hence, the key challenge of handling data with uncertainty in outlier detection which consists of a classifier which is built by considering both normal and abnormal training data. The problem of uncertainty as discussed in previous section is solved by using likelihood values for each record in the dataset. For this, two likelihood models are generated i.e. single and bi likelihood model. For both likelihood models, pseudo training dataset is generated. These two pseudo training datasets are then used for training the support vector data description classifier separately. The contribution of the paper lies in selection of appropriate kernel parameters. Also the proposed system will deal with refinement of the support vector data descriptor classifier boundary. The proposed system architecture is as shown in the Fig.1

B. Uncertain Data
Various techniques have been proposed in order to handle the uncertainty of data in query processing of uncertain data, indexing uncertain data, clustering uncertain data, classification of uncertain data, frequent pattern mining of uncertain data. Work in [22] considers uncertainty in data in the outlier detection problem where a probabilistic definition of outliers in conjunction with density calculation and sampling are used. Different from this work, to handle the problem of outlier detection in the presence of uncertain data, the proposed system proposes a model-based approach by introducing a likelihood value for each input data point into the SVDD training phase. The proposed approach operates in two important steps. In the first step, pseudo-training dataset is generated by assigning a confidence level to each input record, which indicates the likelihood of an input data record belonging to normal class. For this kernel-based clustering method is used to generate the confidence level for each input training record. In the second step, generated confidence score for each sample is incorporated into the SVDD training process. Due to introduction of confidence score into the training stage, each data record contributes differently to the generation of the decision boundary, which then is used for outlier detection.

III. PROPOSED WORK

A. Proposed System:
The proposed system is a systematic approach for outlier detection which consists of a classifier which is built by considering both normal and abnormal training data. The problem of uncertainty as discussed in previous section is solved by using likelihood values for each record in the dataset. For this, two likelihood models are generated i.e. single and bi likelihood model. For both likelihood models, pseudo training dataset is generated. These two pseudo training datasets are then used for training the support vector data description classifier separately. The contribution of the paper lies in selection of appropriate kernel parameters. Also the proposed system will deal with refinement of the support vector data descriptor classifier boundary. The proposed system architecture is as shown in the Fig.1

Following are the modules of the proposed system:

Input Module
Input module includes setting the environment for running the proposed system. This includes initialising the dataset for processing. The module sets the variables and parameters for the
system as per the dataset. Each instance in the d-dataset is represented as X = x₁, x₂, x₃, ..., x₄₉ where x₁, x₂, x₃, ..., x₉ are the features of the dataset.

**Kernel k-means clustering**

In kernel k-means clustering algorithm likelihood, a nonlinear mapping function \( \Phi(.) \) is used to map the input sample into a feature space. For this it needs to minimize the objective function (1):

\[
J = \sum_{i=1}^{k} \sum_{j=1}^{m} || \Phi(x_j) - \Phi(v_i) ||^2
\]

**k:** Number of clusters

\( v_i \): cluster center of \( i \)th cluster

\( m=|X| \)

\( n: \) Total number of normal examples

\( n: \) Total number of abnormal examples

By solving this optimization problem, it returns set of local clusters. Thus, kernel k-means clustering algorithm takes input as the set of data points and the count of number of clusters. It randomly initializes ‘k’ cluster center and compute the distance of each data label point and the cluster center in the transformation space. It then assigns a data point to that cluster whose center distance is minimum. These steps are repeated till data points are reassigned. For a given cluster \( j \), assume that there exist \( l_j \) normal examples and \( l'_j \) negative examples.

**Likelihood value generation function**

**Single likelihood model:**

In this model, each input is associated with a likelihood value \((x_i, m(x_i))\), which represents degree of membership of an example towards its own class label.

**Bi-likelihood model:**

In the model, each input is associate with bi-likelihood values, denoted as \((x_i, m^l(x_i), m^n(x_i))\), in which \(m^l(x_i)\) and \(m^n(x_i)\) indicate the degree of an input data xi belonging to the positive class and negative class respectively.

To calculate the degree of membership value:

**For the single likelihood model,**

\[ m^l(x_i) = l_j / (l_j + l'_j) \]

where \( x_i \) belongs to the normal class

\[ m^n(x_i) = l'_j / (l_j + l'_j) \]

where \( x_i \) belongs to the negative class

For the bi-likelihood model,

\[ m^l(x_i) = l_j / (l^l_j + l^l'_j) \]

where \( x_i \) belongs to the normal class

\[ m^n(x_i) = l'_j / (l^l_j + l^l'_j) \]

**Pseudo training dataset generation**

For the single likelihood model, the generated pseudo training data consists of two parts for the 'l' normal examples and 'n' abnormal examples as follows:

\[(x_1, m^l(x_1)), (x_2, m^l(x_2)), ..., (x_l, m^l(x_l)), (x_{l+1}, m^n(x_{l+1})), ..., (x_{l+n}, m^n(x_{l+n}))\]

where,

\( m^l(x_i) \): likelihood of example \( x_i \) belonging to the normal class

\( m^n(x_i) \): likelihood of example \( x_i \) belonging to the abnormal class

Similarly, the generated pseudo training data for bilikelihood model is:

\[(x_1, m^l(x_1), m^n(x_1)), ..., (x_l, m^l(x_l), m^n(x_l)), (x_{l+1}, m^l(x_{l+1}), m^n(x_{l+1})), ..., (x_{l+n}, m^l(x_{l+n}), m^n(x_{l+n}))\]

**Modified SVDD**

This module is a SVDD classifier which is trained by the pseudo training dataset. SVDD maps the input sample vectors to the kernel space and find a hyper sphere class boundary with the minimum volume as shown in Fig. 2. Modified SVDD is the proposed modified version of SVDD which improves the performance of SVDD by kernel parameter selection and refinement of hyper-sphere class boundary.

**For single likelihood model,**

**Step 1.** All positive examples are put into \( S_p \) set and all negative examples are put into \( S_n \) set. Thus, \( m^l(x_i) \) is degree of membership associated with normal example and \( m^n(x_i) \) is degree of membership associated with abnormal example.

**Step 2.** In order to obtain this hypersphere we need to minimize the objective function (2):

\[
\text{Min } F = \text{R}^2 + C_1 \sum_{i=1}^{l} m^l(x_i) \xi_i + C_2 \sum_{i=1}^{n} m^n(x_i) \xi_j
\]

Such that,

\[
\| x_i - o \|_2^2 \leq \text{R}^2 + \xi_i, \ x_i \ \text{belong to } S_p
\]

\[
\| x_i - o \|_2^2 \geq \text{R}^2 - \xi_i, \ x_i \ \text{belong to } S_n
\]

\[
\xi_i \geq 0, \xi_j \geq 0
\]

Where,

\( R \): Radius of the hypersphere

\( O \): Center of the hypersphere

\( C_1, C_2 \): Parameters that control the tradeoff between the sphere volume and errors

\( \xi_i, \xi_j \): Slack variables

**Step 3.** Use Lagranges method for solving above optimization problem, which includes maximizing the function (3):

\[
\sum_{i=1}^{l} \alpha_i K(x_i, x_i) - \sum_{i=1}^{l} \sum_{j=1}^{n} \alpha_i \alpha_j K(x_i, x_j)
\]

Such that, \( 0 \leq \alpha_i \leq C_i \), \( i = 1, 2, ..., 1 + n \)

\[
\sum_{i=1}^{l} \alpha_i = 1
\]

**Step 4.** After solving the above problem, we obtain Lagrange multiplier \( \alpha_i \) which gives the centroid of the hypersphere by equation (4):

\[
o = \sum_{i=1}^{l} \alpha_i \Phi(x_i)
\]
Step 5. By assuming a point \( x_i \) on the surface of hypersphere, \( R \) can be calculated by equation (5):

\[
R^2 = \|x_n - o\|^2 = K(x_n, x_o) + K(o, o) - 2K(x_n, o) + \sum_{i=1}^{l_{\text{in}}} \sum_{k=1}^{l_{\text{in}}} \alpha_i \alpha_k K(x_i, x_k) - 2 \sum_{i=1}^{l_{\text{in}}} \alpha_i (x_i, x_o)
\]

(5)

Step 6. To classify \( x \), if the distance is less than or equal to \( R \), then it is deemed as normal data as shown in equation (6)

\[
\|x - o\|^2 \leq R^2
\]

(6)

For bi likelihood model,

Step 1. All examples that totally belong to positive class are put into \( S_p \) set, i.e. whose \( m^p(x_i) = 1 \) and \( m^o(x_i) = 0 \). All examples that totally belong to negative class are put into \( S_o \) set i.e. \( m^o(x_i) = 1 \) and \( m^p(x_i) = 0 \). All the remaining examples which neither belong to positive nor negative class are put into \( S_n \) set i.e. \( m^p(x_i) \neq 1 \) and \( m^o(x_i) \neq 0 \). Thus, \( m^o(x_i) \) is degree of membership associated with normal example and \( m^p(x_i) \) is degree of membership associated with abnormal example.

Step 2. In order to obtain this hypersphere we need to minimize the objective function (7):

\[
\text{Min} F = R^2 + C_1 (\sum \xi_i + \sum m^p(x_i) \xi_p) + C_2 (\sum \xi_i + \sum m^o(x_i) \xi_o)
\]

Such that,

\[
\|\Phi(x_n) - o\|^2 \leq R^2 + \xi_p, \quad x_n \text{ belong to } S_p
\]

\[
\|\Phi(x_n) - o\|^2 \leq R^2 + \xi_o, \quad x_n \text{ belong to } S_o
\]

\[
\|\Phi(x_n) - o\|^2 \geq R^2 - \xi_p, \quad x_n \text{ belong to } S_p
\]

\[
\|\Phi(x_n) - o\|^2 \geq R^2 - \xi_o, \quad x_n \text{ belong to } S_o
\]

\[
\xi_p \geq 0, \xi_o \geq 0, \quad \xi_p \geq 0, \xi_o \geq 0
\]

(7)

The meaning of the parameters is same as in single likelihood model.

Step 3. Use Lagrange method for solving above optimization problem, which includes maximizing the function (8):

\[
\text{Max} \sum \alpha^p_i K(x_i, x_n) - \sum \alpha^o_i K(x_i, x_o) - \sum \sum \alpha^p_i \alpha^o_j K(x_i, x_j) + 2 \sum \sum \alpha^o_i \alpha^p_j K(x_i, x_j) + 2
\]

Such that,

\[
0 \leq \alpha^p_i \leq m^p_i(x_i) K_i
\]

\[
0 \leq \alpha^o_i \leq m^o_i(x_i) C_2
\]

\[
\sum \alpha^p_i + \sum \alpha^o_i = 1
\]

\[
x_{i} x_{i} \epsilon S_{p} U S_{o}, x_{i} x_{i} \epsilon S_{p} U S_{o},
\]

Where \( \alpha^p_i \geq 0, \alpha^o_i \geq 0 \) are Lagrange multipliers.

Step 4. After solving the above problem, we obtain Lagrange multiplier \( \alpha \), which gives the centroid of the hypersphere by equation (9):

\[
o = \sum \alpha^p_i \Phi(x_i) + \sum \alpha^o_i \Phi(x_o)
\]

(9)

Step 5. By assuming a point \( x_n \) on the surface of hypersphere, \( R \) can be calculated by equation (10):

\[
R^2 = \|x_n - o\|^2 = K(x_n, x_o) + \sum \sum \alpha^p_i \alpha^o_j K(x_i, x_j) + \sum \sum \alpha^o_i \alpha^p_j K(x_i, x_j) - 2 \sum \sum \alpha^o_i \alpha^p_j K(x_i, x_j) + 2 \sum \sum \alpha^o_i \alpha^p_j K(x_i, x_j)
\]

Step 6. To classify \( x \), if the distance is less than or equal to \( R \), then it is deemed as normal data as shown in equation (11)

\[
\|x - o\|^2 = K(x, x) + \sum \sum \alpha^p_i \alpha^o_j K(x_i, x_j) + \sum \sum \alpha^o_i \alpha^p_j K(x_i, x_j) - 2 \sum \alpha^p_i K(x_i, x) - 2 \sum \sum \alpha^o_i \alpha^p_j K(x_i, x) + 2 \sum \sum \alpha^o_i \alpha^p_j K(x_i, x) \leq R^2
\]

(10)

In order to improve the performance of both SVDD classifiers, the proposed system will select the optimized parameters for the kernel function and will also refine the hypersphere boundary for effective and efficient results. SVDD parameters, \( C \) and \( \sigma \) will massingly influence the accuracy of classification. Here, \( e^{-\|x_i - x\|^2/(2\sigma^2)} \)

(13)

eq. (13) represents the kernel function which is to be optimized. For this, the best pair \( \{C, \sigma\} \) is determined. The ultimate task is to tune the parameters so that the accuracy on the test set and on unseen data is optimal. The optimization problem eq. (2) and (7) will be solved for a given set of parameters \( C \) and \( \sigma \). Determination of parameters is not straightforward. The regularization parameter \( C \), introduced in eq. (2) and (7), is lower-bound by \( 1/N \), where \( N \) is the number of instances in the training data set. \( C = 1 \) corresponds to the hard-margin solution, where all instances are enclosed in the decision boundary. So the value range of \( C \) is \( N \leq C \leq 1 \). The second parameter to be optimized is the kernel width \( \sigma \). For high values of \( \sigma \) the boundary will become spherical with the risk of under fitting, while for small values of \( \sigma \) a high fraction of instances are selected to be support vectors, hence the boundary is very flexible and is prone to over fitting.

**Outlier detection**

This module presents the detected outliers by the previous module and also gives the analysis about the outliers detected.

**IV. EXPERIMENTAL SETUP**

All experimentation is performed using Pentium processor and 4 GB RAM. The operating system is windows 7(32 bit) with visual studio 10.

**A. Dataset:**

A large variety of datasets are available for experimentation such as Abalone, Spambase, Thyroid, Waveform, Satellite, Delft, Diabetes, Segment, Letter and Arrhythmia. For experimentation diabetes dataset is used. Diabetes files consist of glucose measurement pre and post of breakfast, lunch and supper. For normal data the pre-meal glucose measurement is 80 to 120 mg/dl and post-meal glucose measurement is 80-140 mg/dl. If it is less than 40 mg/dl then it is considered as hypoglycemic. And if it is more than 200 then it is considered as hyperglycemic. Diabetes dataset contains 768 patient records.

To perform outlier detection with very few abnormal data, we can randomly select 50% of positive data and a small number of abnormal data for training, such that 95 percent of the training data will belong to the positive class and only 5 percent belong to the negative class. All the remaining data will be used for testing. Experiments are designed to perform 10 fold validations and then further perform cross validation.

**B. Performance Measure:**

In order to evaluate the performance of the system two important performance parameters considered are False Positive Rate and Detection Rate.

This can be depicted as in Table I.
Table VI shows two pseudo training dataset generated for each SVDD classifiers:

<table>
<thead>
<tr>
<th>Patient ID</th>
<th>Single likelihood pseudo training dataset</th>
<th>Bi-likelihood pseudo training dataset</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.833 - 1</td>
<td>0.833 0.167</td>
</tr>
<tr>
<td>2</td>
<td>0.833 - 2</td>
<td>0.833 0.167</td>
</tr>
<tr>
<td>3</td>
<td>- 0.75 3</td>
<td>0.25 0.75</td>
</tr>
<tr>
<td>4</td>
<td>- 0.667 4</td>
<td>0.33 0.667</td>
</tr>
<tr>
<td>5</td>
<td>0.25 - 5</td>
<td>0.25 0.75</td>
</tr>
<tr>
<td>6</td>
<td>0.33 - 6</td>
<td>0.33 0.667</td>
</tr>
<tr>
<td>7</td>
<td>- 0.75 7</td>
<td>0.25 0.75</td>
</tr>
<tr>
<td>8</td>
<td>- 0.75 8</td>
<td>0.25 0.75</td>
</tr>
<tr>
<td>9</td>
<td>0.833 - 9</td>
<td>0.833 0.167</td>
</tr>
<tr>
<td>10</td>
<td>0.833 - 10</td>
<td>0.833 0.167</td>
</tr>
<tr>
<td>11</td>
<td>- 0.167 11</td>
<td>0.833 0.167</td>
</tr>
<tr>
<td>12</td>
<td>0.833 - 12</td>
<td>0.833 0.167</td>
</tr>
<tr>
<td>13</td>
<td>- 0.667 13</td>
<td>0.33 0.667</td>
</tr>
</tbody>
</table>

After testing the dataset against the modified SVDD, we get the set of outliers in the dataset as shown in Table VII.

Table VII. Outliers Detected

<table>
<thead>
<tr>
<th>Patient ID</th>
<th>Pre meal BG</th>
<th>Post meal BG</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>220</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
<td>220</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>135</td>
</tr>
<tr>
<td>12</td>
<td>115</td>
<td>130</td>
</tr>
<tr>
<td>13</td>
<td>106</td>
<td>200</td>
</tr>
</tbody>
</table>

Thus, Fig. 3., depicts the increase in the performance of the proposed system with respect to the existing systems. The performance is calculated by considering the tradeoff between detection rate and false alarm rate.

![Fig. 3. Performance Increase in outlier detection](image)

**CONCLUSION**

Outlier detection is a significant problem with direct application in a wide variety of domains. An important observation with outlier detection is that it is not a well-formulated problem. Several approaches have been proposed to target a particular application domain. A novel approach is introduced to handle outlier mining for unlabeled data. The proposed system is a new model-based approach for outlier detection which introduces likelihood values to each input data into the modified SVDD training phase. The proposed method first captures the local uncertainty by computing likelihood values for each example based on its local data behavior in the feature space, and then builds global classifiers for outlier detection by incorporating the negative examples and the likelihood values in the modified SVDD-based learning framework. Extensive experiments are designed for real-life datasets with performance matrix to measure the performance.
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